Geometric Modeling under Material Constraints

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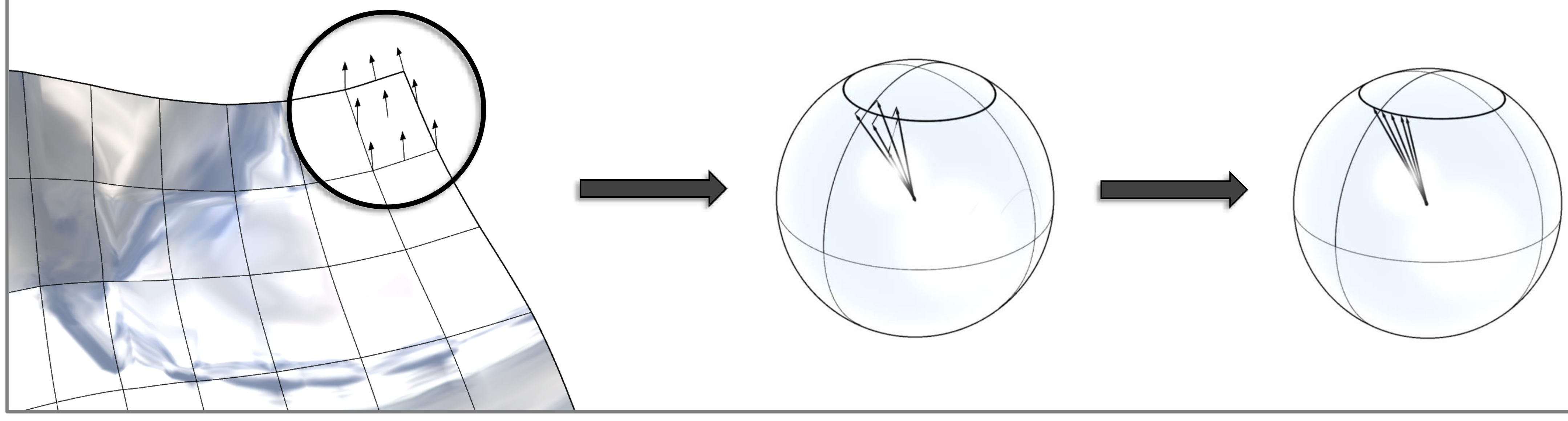
Motivation and Objective. Materials like wood, glass and metal introduce constraints to the abstract geometric modeling of surfaces. We aim at a sufficiently good approximation of the *nearly developable* property that certain materials possess, due to their ability to stretch; an approximation suitable for developing an interactive design system.

Curved glass façade rationalized using Evolute paneling solution.

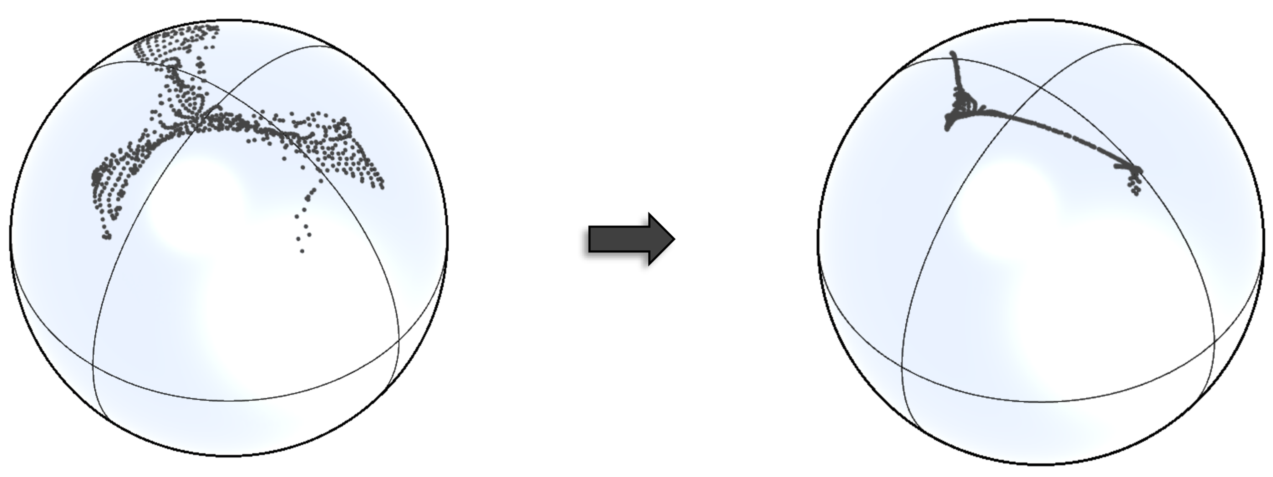
Client: AluTerra Kazakhstan
EXPO 2017 Astana, Kazakhstan



Methodology. By considering neighborhoods P_j (overlapping or not depending on the application scenario), containing points $\boldsymbol{p}_i$ with corresponding normals $\boldsymbol{n}_i$ on the design surface, we optimize for planarity of the neighborhood Gauss images and rotationality of the neighborhoods in the case of surface paneling.

Developability property. Developable surfaces possess a curve as a Gauss image. A "thin" Gauss image concentrated around a curve indicates a nearly developable surface. A spherical curve can be locally well approximated by a planar cut of the sphere as it possesses at every point a circle that has contact of order two (osculating circle).

Example. We optimize for developability a mesh that we obtained from scanning a stretched leather patch.



Initially, the surface had a "fat" Gauss image which indicates that the surface is non-developable.

After the optimization the surface obtains a Gauss image which is "thinner" and approximates a curve.

Increasing developability. Starting with a non-developable surface that has as Gauss image an area on the unit sphere, we use the same optimization approach to achieve a more curve-like Gauss image by considering overlapping neighborhoods on the surface.

Objective function. We optimize having the control points of the panels and the plane H_j intersecting the Gauss image sphere as variables. H_j is given by its unit normal $\boldsymbol{u}_j$ and its distance to the origin d_j.

$$\min \boldsymbol{F} = w_1 \boldsymbol{E}_{developable} + w_2 \boldsymbol{E}_{rotational} + w_3 \boldsymbol{E}_{fairness}$$

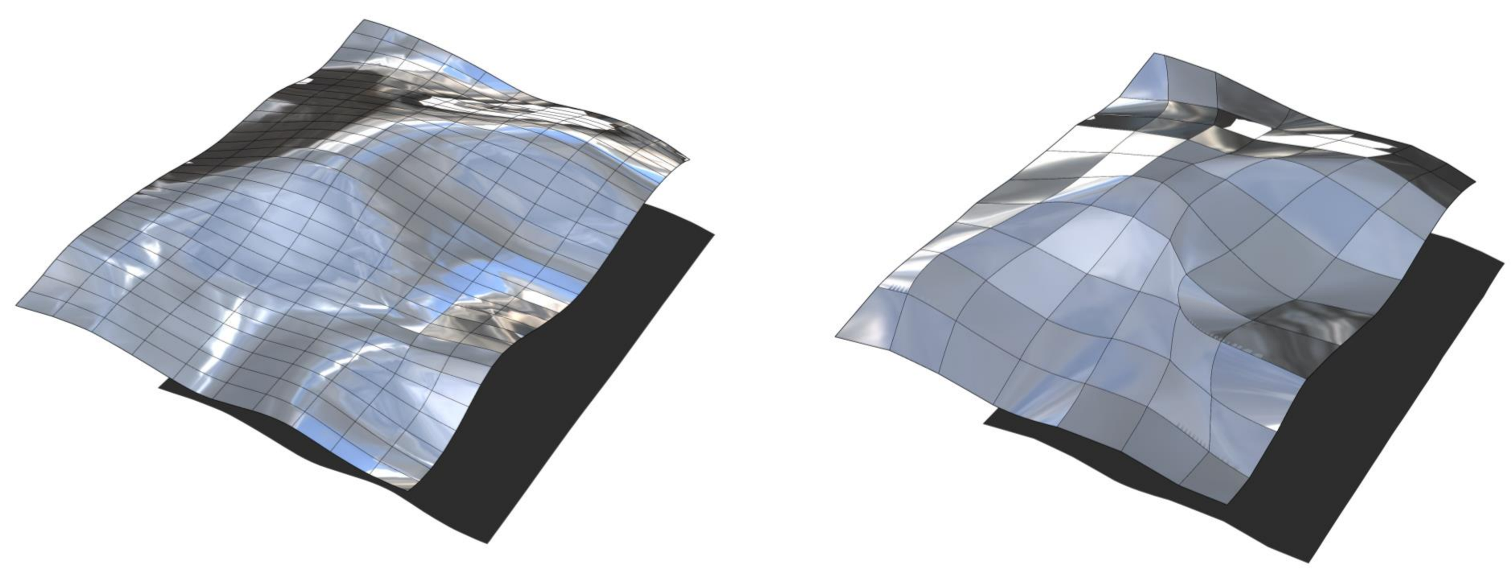
where

$$\boldsymbol{E}_{developable} = \sum_j \sum_{p_i \in P_j} (\boldsymbol{n}_i \cdot \boldsymbol{u}_j + d_j)^2$$

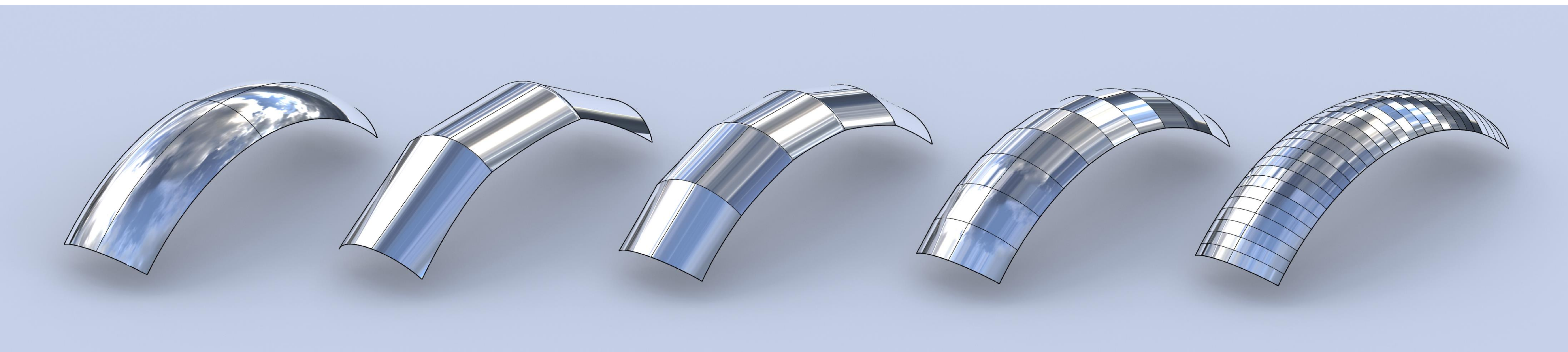
$$\boldsymbol{E}_{rotational} = \sum_j \sum_{p_i \in P_j} (\bar{\boldsymbol{a}}_j \cdot \boldsymbol{n}_i + \boldsymbol{a}_j \cdot \bar{\boldsymbol{n}}_i)^2$$

subject to

$$\bar{\boldsymbol{a}}_j{}^2 + \boldsymbol{a}_j{}^2 = 1, \qquad \boldsymbol{n}_i{}^2 = 1, \qquad \boldsymbol{u}_j{}^2 = 1$$



Applications. Approximating a reference design surface with a panelized surface consisting entirely of circular cylindrical, circular conical and planar panels considerably reduces the cost of manufacturing and complexity of construction. Immediate applications are in areas such as architecture, engineering and design.



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